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# Analysis of the calving pattern of herds in small-scale dairy systems in central Mexico*

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ABSTRACT

This study sets out to find the best calving pattern for small-scale dairy systems in Michoacan State, central Mexico. Two models were built. First, a linear programming model was constructed to optimize calving pattern and herd structure according to metabolizable energy availability. Second, a Markov chain model was built to investigate three reproductive scenarios (good, average and poor) in order to suggest factors that maintain the calving pattern given by the linear programming model. Though it was not possible to maintain the optimal linear programming pattern, the Markov chain model suggested adopting different reproduction strategies according to period of the year that the cow is expected to calve. Comparing different scenarios, the Markov model indicated the effect of calving interval on calving pattern and herd structure.

KEY WORDS: small-scale dairy systems, calving pattern, linear programming, Markov chain

## INTRODUCTION

The basic economic goal in managing a dairy enterprise is to use available resources in order to maximize profit, or at least achieve the goal over the long term. Mulholland (2001) reported the benefits obtained from tightening

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the calving pattern to take better account of labour and forage availability. However, the most relevant benefit in the central Mexican context relates to obtaining a steady cash flow by avoiding sudden fluctuations in milk production over the year.

Specific studies of calving patterns in Mexican dairy systems are non-existent, though a few reports have given the topic a brief mention. Candler (1983) attempted to build a linear programming (LP) model of an intensive dairy farm in northern Mexico that takes into account all the resources of the farm and then suggests an optimal herd structure over the year. However, the project was not finished for administrative reasons. Castro-Lopez et al. (2001) reported that seasonality of milk production is due to higher forage availability during the rainy season but failed to mention whether calving pattern was related to this seasonality. Castelan-Ortega et al. (2003) developed a model to simulate milk production on small-scale dairy farms in the Toluca Valley in central Mexico, considering resources such as number of cows each month and forage availability. Based on the results from this model, they suggested that calving should be during the rainy season to take advantage of higher forage availability. However, their model was not built to determine specifically an optimal calving pattern and herd structure.

Several models have been developed to analyse the effects of different replacement policies, using a Markov chain (MC) approach in combination with LP. These models have offered different degrees of complexity depending on the objectives defined and the availability of information (e.g., Ben-Ari et al., 1983; Yates and Rehman, 1998; Mouritis et al., 1999). However, only a few models have been developed to determine an optimal calving pattern. Keane (1991) built a model at the sector level to suggest the optimal pattern by considering a quota system of milk supply from Irish dairy farms. Jalvingh et al. (1993a) developed a model that optimizes the calving pattern using a MC approach. However, the MC approach by itself cannot account for inherent constraints on dairy farms in terms of management capability and feed availability, among others. Jalvingh et al. (1994) subsequently proposed a more complex model combining both MC and LP approaches in which farm and herd restrictions were taken into account, although these constraints were only considered theoretically.

The prime objective of this study was to develop a combination of a LP and a MC model that allows an improved calving pattern for small-scale dairy systems in central Mexico to be identified without demanding detailed information. This combination of models permits both the restrictions on the farm and the dynamics of the herd to be taken into account.

## MODEL DESCRIPTION

### *General*

The LP and MC models were developed by taking into account both the lack of and the difficulty in collecting technical information for the kind of dairy enterprise considered in this study. Therefore, the models were constructed using only the most relevant information needed to achieve the prime objective. The study was accomplished in two phases. The first phase was to develop an LP model to find an optimal herd structure and calving pattern by maximizing milk production over the year subject to metabolizable energy (ME) availability and capacity constraints of the farm. The second phase was to integrate the LP results within a MC model of herd dynamics to investigate three reproductive scenarios in order to suggest factors that maintain the optimal calving pattern and herd structure given by the LP model.

### *LP model*

The model describes a small-scale dairy system with a herd of 13 cows, each with an average liveweight of 550 kg, an average milk yield of approximately 14 kg day$^{-1}$, and a lactation of 305 day. The ME supply is from a typical forage production strategy based on fresh cut lucerne, maize grain and maize straw (Cazarin et al., 2000). The model has a time frame of one calendar year divided into four quarters of three months each. The lactation cycle was also divided into four equal stages of 91 days: calving (comprising a dry period of 59 days plus the first 32 days of lactation), second stage, third stage and last stage. Thus, the herd structure in each calendar quarter consists of calving cows (C), cows in the second (2S), in the third (3S) and in the last stage (LS) of lactation. The algebraic statement of the LP model is:

$$\text{maximize: } y = \sum_{i=1}^{4} a_i \sum_{j=1}^{4} x_{i,j},$$

subject to:

$$\sum_{i=1}^{4} c_i x_{i,j} \le u_j, \quad j = 1,2,3,4,$$

$$\sum_{i=1}^{4} x_{i,j} = h, \quad j = 1,2,3,4,$$

$$x_{i+1,j+1} \leq (1-p) \times x_{i,j}, \quad i = 1,2,3, \quad j = 1,2,3,$$
$$x_{1,j+1} \leq (1-p) \times x_{4,j}, \quad j = 1,2,3,$$
$$x_{i,j} \geq 0, \quad i = 1,2,3,4, \quad j = 1,2,3,4,$$

where:
$y$ = annual milk yield (kg)
$x_{i,j}$ = number of cows in stage $i$ of their lactation cycle in calendar quarter $j$
$a_i$ = milk yield of a cow in stage $i$ of lactation (kg)
$c_i$ = ME requirement of a cow in stage $i$ of lactation (MJ)
$u_j$ = availability of ME in quarter $j$ (MJ)
$h$ = herd size (which is taken to be the same in each quarter)
$p$ = probability of mortality per quarter.

The model deals with variation in forage supply (in terms of ME availability) by optimizing the calving pattern and therefore the herd structure over the year according to ME availability in each quarter. Thus, the model seeks to allocate cows in lactation stages with higher ME requirement to calendar quarters with higher ME availability.

The milk produced by a cow in a specific stage of lactation was calculated from the model by Dijkstra et al. (1997). A typical 305-day lactation curve for a cow in a small-scale dairy system in central Mexico was assumed in fitting the Dijkstra equation, then milk yield over each lactation period was cumulated using the numbers of days mentioned previously (i.e. 32, 91, 91 and 91).

The daily ME requirements were estimated according to the recommendations of AFRC (1993). To calculate the total amount of energy required for a cow in specific stage of its lactation cycle, the appropriate daily ME requirements were summed. A negative energy balance was assumed for 1 month.

The ME availability over the year from a traditional forage production strategy of fresh cut lucerne, maize grain and maize straw was taken from Val-Arreola et al. (2002). The daily availability per harvesting cycle (spring-summer and autumn-winter) determined by these authors was converted to a monthly basis, and summed to obtain the energy available in each quarter. For example, the energy available in January, February and March were added to obtain the ME upper limit for the first quarter.

Herd size of 13 cows was taken as the typical herd size reported by Val-Arreola et al. (2002). A value of 2.5% per annum was used for the probability of mortality in each quarter (Candler, 1983). Other values used in the LP model are shown in Table 1. The LP model was solved as an ordinary linear programme using a revised simplex algorithm.

TABLE 1
LP coefficients for stage of lactation ($i$ = 1, 2, 3, 4) and right hand side values for calendar quarter ($j$ = 1, 2, 3, 4)

| | Stage of lactation or calendar quarter | | | | |
|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | Total |
| $a_i$, kg | 231 | 1,242 | 1,251 | 976 | 3,700 |
| $c_i$, MJ | 1,920 | 12,420 | 12,328 | 14,454 | 41,122 |
| $u_j$, MJ | 182,470 | 104,935 | 104,935 | 221,238 | 613,578 |

*MC model*

A Markov chain is a discrete stochastic process in which the probability distribution of state at time ($i$ + 1) depends on the state at time $i$ and not on previous states at time ($i$ – 1), ($i$ – 2), etc. Although not an optimization method, this is a useful tool that allows the behaviour of a system to be analysed. Models applying MC have been used in dairy production, for example, to determine policies for insemination and replacement (Kristensen, 1991) and to evaluate the potential effects of embryo transfer on milk production (Yates et al., 1996). In the present study, the MC approach is used to provide an insight into the consequences of decisions taken to realise the optimum calving pattern suggested by the LP model.

The model was constructed assuming that the amount and quality of information on a typical small-scale dairy enterprise is not sufficient to build a detailed transition matrix. Therefore, general reproduction coefficients and the main framework of the LP were used to construct a simple transition matrix that allows evaluation of the possible effects of certain reproduction policies on the calving pattern and herd structure suggested by the LP. As previously, the model has a time frame of one calendar year divided into four quarters of three months each and a lactation cycle divided into four equal stages of 91 days: calving cows (C), cows in the second stage (2S), cows in the third stage (3S) and cows in the last stage (LS) of lactation.

To construct the transition matrix, it was assumed that a cow in going from one calendar quarter to the next either remains in the same stage of lactation or moves to the next stage. The cows that calved in the present quarter are taken to remain in the same stage of lactation in going to the next quarter. The same assumption is made for replacements. The others move to the next stage. The following equations were used to compute the proportions $P_1$ and $P_2$:

$$X_1 = \left[\left(3 \times \frac{C_a}{12}\right) \times (1 - M)\right] + R,$$

$$X_2 = \left(9 \times \frac{C_a}{12}\right) \times (1 - M),$$

$$C_a = \left(\frac{C}{I}\right) \times 30.5 \times 12,$$

$$P_1 = \frac{X_1}{X_1 + X_2},$$

$$P_2 = \frac{X_2}{X_1 + X_2},$$

where:

$P_1$ = proportion of cows that remain in same stage of lactation when going from one calendar quarter to the next

$P_2$ = proportion of cows that move to the next stage of lactation when going from one calendar quarter to the next

$C_a$ = adjusted calving rate (adjusted to an annual basis)

$M$ = quarterly mortality rate

$R$ = quarterly replacement rate

$C$ = actual calving rate

$I$ = calving interval (days)

and each rate is expressed as a proportion of herd size (i.e. in units of cows cow$^{-1}$).

The equations assume constant calving, mortality and replacement rates in each quarter of the year. The proportions $P_1$ and $P_2$ are the constants in the transition matrix **P** shown below:

$$\text{FROM} \quad \begin{matrix} \text{C} \\ \text{2S} \\ \text{3S} \\ \text{LS} \end{matrix} \quad \mathbf{P} = \overset{\text{TO}}{\overset{\begin{matrix} \text{C} & \text{2S} & \text{3S} & \text{LS} \end{matrix}}{\begin{bmatrix} P_1 & P_2 & 0 & 0 \\ 0 & P_1 & P_2 & 0 \\ 0 & 0 & P_1 & P_2 \\ P_2 & 0 & 0 & P_1 \end{bmatrix}}}$$

Having constructed the transition matrix, the herd structure in successive quarters was calculated using the following iterative statement: $\mathbf{x}_n = \mathbf{x}_{n-1}\mathbf{P}$. The solution to the LP model was used as the starting value $\mathbf{x}_0$. The simulation covered a period of one year and considered the three scenarios described below, allowing the impact of different policies on the dynamics of herd structure and herd performance to be evaluated.

*MC scenarios and coefficients*

Three alternative scenarios were used to evaluate the impact of different reproduction management policies on the calving pattern and herd structure resulting from the LP model (Table 2). The scenarios had average (Scenario I), good (Scenario II) and poor (Scenario III) reproduction parameters. The calculation of adjusted calving rate is based on Esslemont and Kossaibati (1995). Given the small size of the herd, the mortality rate was assumed to be 2.5% per annum, the value utilized by Candler (1983) in his LP model for Mexican dairy herds.

TABLE 2

Reproduction coefficients used in the MC model

| Coefficient | Scenario | | |
|---|---|---|---|
| | I | II | III |
| Calving interval, days | 380[a] | 368[b] | 400[c] |
| Calving rate, % | 78[a] | 86[b] | 69[c] |
| Replacement rate, % | 22[d] | 16[b] | 30[c] |

[a] Ramirez-Gonzalez (2002)
[b] Esslemont and Kossaibati (1995)
[c] data from sample of small-scale dairy farms in Michoacan State
[d] Wiggins et al. (2001)

For Scenario I, the calving interval was taken from studies carried out on 12 small-scale dairy farms in Michoacán State by Ramírez-González (2002), who found on average a 380 day calving interval, 130 days open and 1.4 services per conception. Earlier studies report not too dissimilar parameters (a 424 day calving interval, 127 days open, 1.5 services per conception), but also mention that 55% of the herd observed a deficient reproductive efficiency that might be related to a low energetic balance (Salas-Razo, 1998). The calving rate was taken as the average obtained from the information collected on the 12 farms. The replacement rate was set at 22%, based on six lactations per cow reported by Wiggins et al. (2001) for small-scale dairy herds in central Mexico (Table 2). For Scenario II, the coefficients were taken from Esslemont and Kossaibati (1995). For Scenario III, the coefficients were taken from three farms with the poorest reproductive performance of the 12 sampled by Ramírez-González (2002). The values of calving interval, calving rate and replacement rate are shown in Table 2.

## RESULTS

### *LP model*

Table 3 shows the results from the LP model. Comparing ME availability and ME consumption, it is possible to cover the energy demand of the herd in all the quarters. However, energy availability during the second and third quarters is tight, as the solution gives the same availability and consumption.

TABLE 3

Results from the LP model for each quarter

| | Quarter | | | |
|---|---|---|---|---|
| | 1 | 2 | 3 | 4 |
| ME availability, MJ | 182,470 | 104,935 | 104,935 | 221,238 |
| ME consumption, MJ | 164,957 | 104,935 | 104,935 | 159,121 |
| Calving stage[1] | 0.1 | 7.3 | 6.6 | 0.0 |
| Second stage[1] | 0.0 | 0.1 | 6.4 | 6.6 |
| Third stage[1] | 6.6 | 0.0 | 0.0 | 6.4 |
| Last stage[1] | 6.4 | 5.7 | 0.0 | 0.0 |
| Milk yield, L | 14,092 | 10,099 | 12,557 | 16,274 |

[1] number of cows

The model allocates calving to the second and third quarters, though these are the periods with less energy available (Figure 1). The highest energy consumption occurs in the first and fourth quarters. Such a situation is reflected in the quarterly milk yields. The highest yields are obtained in the first and fourth quarters, which also have the highest number of cows in the second, third and last stages of lactation (Table 3).

Milk production determined by the model and that observed on small-scale dairy farms in Michoacan State can be compared by contrasting Figures 1 and 2. The milk yields in both figures are in broad agreement. The calving pattern observed on small-scale farms and that generated by the model both show calving concentrated within a six month period. In the case of the model, calving was during the second and third quarters, whereas for the small-scale farms the calving is largely during the third and fourth quarters.

Table 4 summarizes the sensitivity analysis on the constraints for the optimal LP solution. This summary shows the values of the constraints, the shadow prices and allowable increase and decrease values. As the objective function maximizes milk production (53,023 litres) according to ME availability and the

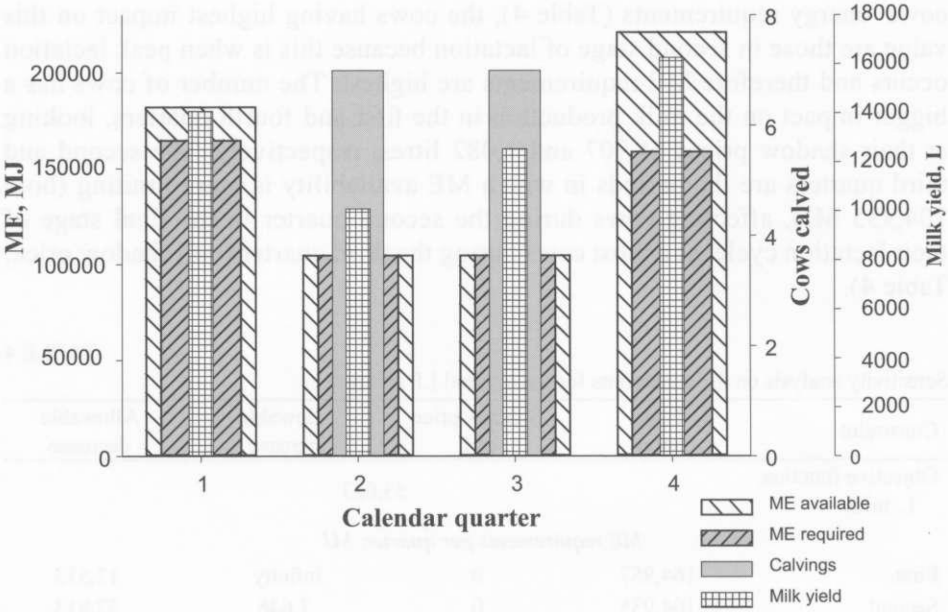


Figure 1. Metabolizable energy, milk yield and cows calved for each quarter as determined by the linear programming model

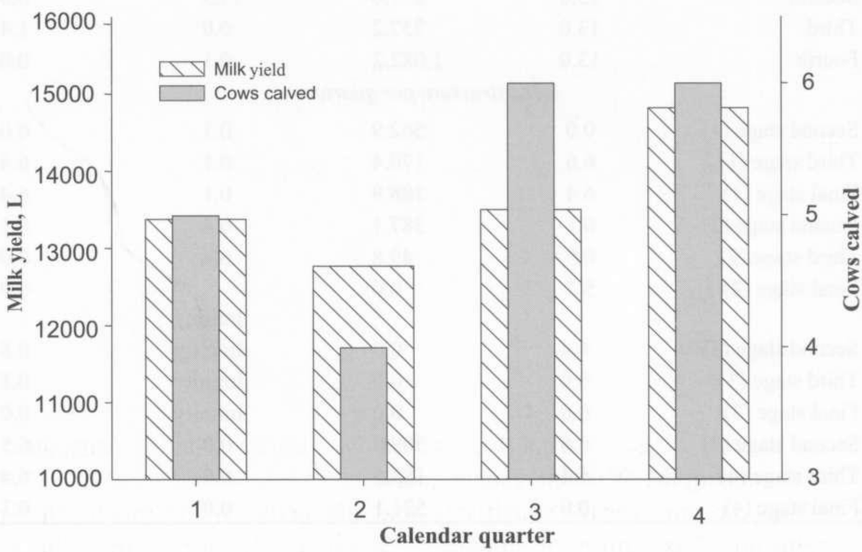


Figure 2. Observed milk yield and calving pattern for each quarter for a typical Mexican small-scale dairy herd (Ramirez-Gonzalez, 2002)

cows' energy requirements (Table 4), the cows having highest impact on this value are those in second stage of lactation because this is when peak lactation occurs and therefore ME requirements are highest. The number of cows has a bigger impact on the milk production in the first and fourth quarters, looking at their shadow prices of 907 and 1,082 litres, respectively. The second and third quarters are the periods in which ME availability is more limiting (both 104,935 MJ), affecting cows during the second quarter in the final stage of their lactation cycle, and most cows during the third quarter (null shadow price; Table 4).

TABLE 4

Sensitivity analysis on the constraints for the optimal LP solution

| Constraint | Final value | Shadow price of L | Allowable increase | Allowable decrease |
|---|---|---|---|---|
| Objective function, L milk | | 53,023 | | |
| | | *ME requirements per quarter, MJ* | | |
| First | 164,957 | 0 | Infinity | 17,513 |
| Second | 104,935 | 0 | 7,646 | 52,813 |
| Third | 104,935 | 0 | 6,790 | 53,351 |
| Fourth | 159,121 | 0 | Infinity | 62,117 |
| | | *Number of cows per quarter* | | |
| First | 13.0 | 907.3 | 4.4 | 0.1 |
| Second | 13.0 | 277.6 | 13.3 | 0.6 |
| Third | 13.0 | 757.2 | 0.0 | 1.4 |
| Fourth | 13.0 | 1,082.2 | 0.1 | 0.0 |
| | | *Herd structure per quarter* | | |
| Second stage (1) | 0.0 | 562.9 | 0.1 | 0.0 |
| Third stage (1) | 6.6 | 170.4 | 0.1 | 6.4 |
| Final stage (1) | 6.4 | 188.9 | 0.1 | 6.4 |
| Second stage (2) | 0.1 | 387.1 | 6.4 | 0.1 |
| Third stage (2) | 0.0 | 49.8 | 6.4 | 0.0 |
| Final stage (2) | 5.7 | 0.0 | | 0.9 |
| | | | Infinity | |
| Second stage (3) | 6.4 | 0.0 | Infinity | 0.8 |
| Third stage (3) | 0.0 | 0.0 | Infinity | 0.1 |
| Final stage (3) | 0.0 | 0.0 | Infinity | 0.0 |
| Second stage (4) | 6.6 | 509.0 | 0.0 | 6.5 |
| Third stage (4) | 6.4 | 183.5 | 0.0 | 6.4 |
| Final stage (4) | 0.0 | 521.1 | 0.0 | 0.1 |

*MC model*

Table 5 shows results from the MC model using the herd structure for the first quarter proposed by the LP model as the initial structure. It can be seen that after simulating for one year, most of the cows are in their second or third stage of lactation. Each scenario shows similar distributions, though a slightly higher portion of the herd is in the second and third stages of lactation for Scenarios I and II (average and good, respectively) than for Scenario III (poor). Results suggest that if the reproduction coefficients given in Table 2 are kept constant all year round, the herd structure shows some similarity to the structure for the fourth quarter proposed by the LP model, in which the cows are in the second and third stages of lactation.

TABLE 5

Results from the MC model of simulating the herd structure for a year using the first quarter values suggested by the LP model as the initial structure

| Quarter | Herd structure | | | |
|---|---|---|---|---|
| | C | 2S | 3S | LS |
| | Initial structure | | | |
| | 0.1 | 0.0 | 6.6 | 6.4 |
| | *Scenario I* | | | |
| 1 | 4.5 | 0.1 | 2.0 | 6.5 |
| 2 | 5.9 | 3.1 | 0.6 | 3.3 |
| 3 | 4.1 | 5.1 | 2.4 | 1.5 |
| 4 | 2.3 | 4.4 | 4.3 | 2.1 |
| | *Scenario II* | | | |
| 1 | 4.6 | 0.1 | 1.9 | 6.5 |
| 2 | 6.0 | 3.3 | 0.6 | 3.2 |
| 3 | 4.0 | 5.2 | 2.5 | 1.3 |
| 4 | 2.1 | 4.3 | 4.4 | 2.2 |
| | *Scenario III* | | | |
| 1 | 4.3 | 0.1 | 2.2 | 6.5 |
| 2 | 5.8 | 2.9 | 0.8 | 3.6 |
| 3 | 4.3 | 4.8 | 2.2 | 1.7 |
| 4 | 2.6 | 4.5 | 4.0 | 2.0 |

Table 6 shows results of simulating the herd structure for a year using the second quarter values from the LP solution as the initial structure. A slightly higher portion of the herd is in the third and last stages of lactation for Scenarios I and II than for III. Herd structure shows some similarity to structure for the first quarter given by the LP. When LP values for the third quarter are used for initial structure, most cows are in the first and last stages of lactation after one year (Table 7). Comparing scenarios, a slightly higher portion of the herd is in the calving and last stages of lactation for Scenarios I and II. Table 8 shows results when LP values for the fourth quarter are used as initial structure. The herd shows some similarity in structure to that for the third quarter given by the LP. Comparing scenarios, a slightly higher portion of the herd is in the calving and second stages of lactation for Scenarios I and II.

TABLE 6

Results from the MC model of simulating the herd structure for a year using the second quarter values suggested by the LP model as the initial structure

| Quarter | Herd structure | | | |
|---|---|---|---|---|
| | C | 2S | 3S | LS |
| | Initial structure | | | |
| | 7.3 | 0.1 | 0.0 | 5.7 |
| | *Scenario I* | | | |
| 1 | 6.2 | 5.1 | 0.0 | 1.7 |
| 2 | 3.1 | 5.8 | 3.6 | 0.5 |
| 3 | 1.3 | 3.9 | 5.2 | 2.7 |
| 4 | 2.3 | 2.1 | 4.3 | 4.4 |
| | *Scenario II* | | | |
| 1 | 6.1 | 5.2 | 0.0 | 1.6 |
| 2 | 2.9 | 5.9 | 3.8 | 0.5 |
| 3 | 1.2 | 3.7 | 5.3 | 2.8 |
| 4 | 2.4 | 1.9 | 4.2 | 4.6 |
| | *Scenario III* | | | |
| 1 | 6.2 | 4.9 | 0.0 | 1.9 |
| 2 | 3.3 | 5.8 | 3.3 | 0.6 |
| 3 | 1.5 | 4.1 | 5.0 | 2.4 |
| 4 | 2.1 | 2.4 | 4.4 | 4.1 |

The MC model generally tends to give a herd structure after one year that is not dissimilar to the structure proposed for the previous quarter by the LP model, suggesting that the cows will take more than a year to calve again. The possibility of returning to the optimal calving pattern suggested by the LP model is reduced with Scenario III. However, factors such as disease and fertility problems need to be incorporated into the model if more accurate results are to be achieved.

TABLE 7

Results from the MC model of simulating the herd structure for a year using the third quarter values suggested by the LP model as the initial structure

| Quarter | Herd structure | | | |
|---|---|---|---|---|
| | C | 2S | 3S | LS |
| | Initial structure | | | |
| | 6.6 | 6.4 | 0.0 | 0.0 |
| | *Scenario I* | | | |
| 1 | 2.0 | 6.5 | 4.5 | 0.0 |
| 2 | 0.6 | 3.4 | 5.9 | 3.1 |
| 3 | 2.4 | 1.4 | 4.1 | 5.1 |
| 4 | 4.3 | 2.1 | 2.2 | 4.4 |
| | *Scenario II* | | | |
| 1 | 1.9 | 6.5 | 4.6 | 0.0 |
| 2 | 0.5 | 3.2 | 6.0 | 3.3 |
| 3 | 2.5 | 1.3 | 4.0 | 5.2 |
| 4 | 4.5 | 2.2 | 2.1 | 4.3 |
| | *Scenario III* | | | |
| 1 | 2.2 | 6.5 | 4.3 | 0.0 |
| 2 | 0.7 | 3.6 | 5.8 | 2.9 |
| 3 | 2.2 | 1.7 | 4.3 | 4.8 |
| 4 | 4.0 | 2.0 | 2.6 | 4.5 |

TABLE 8

Results from the MC model of simulating the herd structure for a year using the fourth quarter values suggested by the LP model as the initial structure

| Quarter | Herd structure | | | |
|---|---|---|---|---|
| | C | 2S | 3S | LS |
| | Initial structure | | | |
| | 0.0 | 6.6 | 6.4 | 0.0 |
| | *Scenario I* | | | |
| 1 | 0.0 | 2.0 | 6.5 | 4.5 |
| 2 | 3.1 | 0.6 | 3.4 | 5.9 |
| 3 | 5.1 | 2.4 | 1.4 | 4.1 |
| 4 | 4.4 | 4.3 | 2.1 | 2.2 |
| | *Scenario II* | | | |
| 1 | 0.0 | 1.9 | 6.5 | 4.6 |
| 2 | 3.3 | 0.5 | 3.2 | 6.0 |
| 3 | 5.2 | 2.5 | 1.3 | 4.0 |
| 4 | 4.3 | 4.5 | 2.2 | 2.1 |
| | *Scenario III* | | | |
| 1 | 0.0 | 2.2 | 6.5 | 4.3 |
| 2 | 2.9 | 0.7 | 3.6 | 5.8 |
| 3 | 4.8 | 2.2 | 1.7 | 4.3 |
| 4 | 4.5 | 4.0 | 2.0 | 2.6 |

## DISCUSSION

A full and independent comparison of the results from the models presented herein was not possible due to a lack of appropriate studies in the Mexican context. When the results from the LP model were compared with the calving pattern and milk yield observed on small-scale dairy enterprises in Michoacan State, central Mexico, the LP solution was found to differ from the observed calving pattern. However, milk production showed a similar trend over the four quarters, though the milk volume estimated by the LP model was slightly higher (Figures 1 and 2). Generally, milk production in Mexico shows a seasonal pattern that is higher during the third quarter of the year because forage availability and quality tend to increase in the rainy season (Castro-Lopez et al., 2001). The LP model mimicked this element reasonably well (Figures 1 and 2). The higher volume estimate is a result of allocating cows optimally by considering ME availability and the cows' lactation stage.

Castelan-Ortega et al. (2003) simulated maize production and the response of cows to feed. They found an increase in milk yield during the rainy season (52% more than in the dry season), which occurs in the third and beginning of the fourth quarter of the year. The authors assumed that during this period there are more cows calving and lactating than during the rest of the year, in agreement with the results obtained from the LP model. However, the results from the model will depend on the system of forage production adopted by the farmer, since each system will provide a different amount of energy at different periods of the year, according to the characteristics of forage itself and the possibility of acquiring external inputs for feeding to the cattle. Also, as the LP model does not take direct account of management quality, the LP solution would be difficult to adopt if reproduction management is substandard. It is evident that calved cows in early and mid-lactation consume more energy. It is also evident that in some periods of the year there is surplus feed and in others there is a deficit. This suggests forage strategies be adopted that allow the use of conservation methods, thus permitting transfer of surplus feed to periods of deficit or little slack.

Results from the MC model with different reproduction scenarios, though not highly accurate, show the effect calving interval has on calving pattern and herd structure. Esslemont et al. (1985) stated it is possible modify calving pattern and herd structure by changing calving rate and replacement rate. Therefore, recommendations made by DeLorenzo et al. (1992) about the implementation of techniques such as synchronized insemination according to optimal herd structure and replacement decisions based on the optimal calving pattern could be appropriate for this type of dairy enterprise. However, Jalvingh et al. (1993a,b) questioned such an approach because sometimes the availability of replacements may not be enough to meet the needs of the enterprise as it depends on the calving pattern in previous years. Esslemont et al. (1985) on the other hand pointed out that longer calving intervals have a minor effect on low yielding cows. Ramírez-González (2002) mentioned that in case of small-scale dairy enterprises the relatively long calving intervals depend on a voluntary waiting period, which allows farmers to schedule the calving period according to the milk demand. Our model can assist in decision taking by determining the optimal waiting period according to milk requirement and energy availability. Nevertheless, the real impact of adopting a particular reproductive strategy depends on economic performance achieved, especially in the long term, but such analysis is outside the scope of the present study.

Although the optimal calving pattern and the herd structure suggested by the LP model agree with other studies, it was not possible to determine the most suitable reproduction scenario to achieve such targets. However, the results obtained from the LP and MC models suggest adopting different reproduction management strategies in order to allocate calving to the most appropriate periods of the year. This means is might be better for the farmer work with different sets of reproduction coefficients (probability two), rather than utilize a single set over the whole year, that take into account the minimum level of milk required, the months in which the demand for milk is highest, and the months with highest and lowest energy availability.

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STRESZCZENIE

**Analiza przebiegu wycieleń w stadach krów mlecznych o małej skali produkcji w środkowym Meksyku**

Celem pracy było określenie optymalnego modelu opisującego przebieg wycieleń krów mlecznych w stadach o małej skali produkcji w Stanie Michoacan w środkowym Meksyku. Zbudowano dwa modele. Pierwszy z nich - model programowania liniowego - został skonstruowany do optymalizacji przebiegu wycieleń i struktury stada zgodnie z dostępnością energii metabolicznej. Drugi z modeli - model łańcuchów Markova został skonstruowany do oceny trzech typów rozrodczości (dobrej, średniej, słabej), celem wskazania czynników determinujących przebieg wycieleń opisany przez model programowania liniowego. Chociaż nie było możliwe uzyskanie optymalnej formuły programowania liniowego, model łańcuchów Markova proponował przyjęcie różnych strategii rozrodu w zależności od sezonu wycielenia. Porównując różne typy wycieleń, na podstawie modelu Markova wskazano na potrzebę uwzględnienia wpływu długości okresu międzywycieleniowego na przebieg wycieleń i strukturę stada.